The Effect and Analysis of Cayley-Menger Determinant on Coordinates Determination of Sensors

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Abstract

In this paper, we study the localization problem in large-scale underwater sensor networks. We present a method to calculate the sensor node coordinate using single floating beacon node. In underwater wireless sensor networks to determine the specific coordinate of the sensors that collect data is essential. The collected data gives limited value into the actual origin. Trilateration or multilateration techniques is used to determine the location of the sensors with respect to more known beacon nodes and measure the distance between sensor and beacon using acoustic signal(considered the roundtrip time).This method do not measure correct distance between sensor and beacon **.** In this study, we determined the underwater distances between beacon and sensor nodes using combined radio and acoustic signals, which has better immunity from multipath fading. Cayley-Menger determinant is used to determine the coordinates of the nodes. The simulations and field evaluations show a good estimation of the beacon node positions. Computing coordinates of sensor nodes with negligible errors.

Declaration

We hereby declare that, this project was done under CSE497 and has not been submitted elsewhere for requirement of any degree or diploma or for any purpose except for publication.

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Letter of Acceptance

We hereby declare that this thesis is from the student's own work and best effort of mine, and all other source of information used have been acknowledge. This thesis has been submitted with our approval.

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Abbreviation and Acronyms

UWSN=Underwater Wireless Sensor Network

GPS= Global Positioning System

NDLP= Node Discovery and Localization Protocol

ALS= Area Localization Scheme

MLSL= Maximum-likelihood Source Localization Approach

LPS= Local Positioning System

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Chapter 1

Introduction

1.1 About underwater wireless sensor networks (UWSNs)

Underwater wireless sensor network is a very prominent and interesting research sector in recent years. One important reason is that they can improve ocean exploration and fulfill the needs of a multitude of underwater applications, including: oceanographic data collection, warning systems for natural disasters (e.g., seismic and tsunami monitoring), ecological applications (e.g., pollution, water quality , food, medicine, and raw materials, marine ecology, military underwater surveillance, assisted navigation, industrial applications (offshore exploration)[1], monitoring aqueous environments for scientific exploration, commercial exploitation, protection from attack [1] etc. Many application use underwater sensors for measure data, the measurement of data are meaningless without knowing the location from where the data are obtained. In addition to underwater sensor nodes, the network may also comprise surface stations and Autonomous Underwater Vehicles (AUVs). Regardless of the type of deployment (outdoor, indoor , underground or underwater), the location of the sensors needs to be determined for meaningful interpretation of the sensed data[2]. Hence, message exchanges between submerged UWSN nodes and surface nodes needed for localization must be carried out, usually using acoustic communications. Unfortunately, underwater acoustic channels are characterized by long propagation delays, limited bandwidth, motion-induced Doppler shift, multipath interference, etc[2].

1.2 Which Problem face in UWSN localization technology

Acoustics is a proven technology for underwater sensor applications which offers long transmission ranges of up to 20 km [3], radio signal transmission range 1.8 to 323[6], although certain challenges and limitations have also been revealed [4, 5].

1.3 Techniques of solving UWSN localization

There are few techniques to solve the problem. In this paper we focus on 3D coordinates detection in under water sensors for all the sensor nodes by measuring distances between beacon and them using Cayley-Menger determinant. In chapter 3 we discussed about Cayley-Menger determinant.

1.4 Objective of the Research

3D Euclidean distance estimation method requires the need of a certain number of neighboring nodes to measure inter-node distances. we assumes at least three submerged sensors deployed in the water and a floating beacon, also measuring the multiple distances between the beacon and sensors, those locations of the beacon are assumed to be in a plane, which is approximately parallel to the plan created by the three sensors. To obtain the accurate coordinates of all the sensor nodes by measuring distances between beacon and sensor nodes is the objective of localization algorithms . To solve this kinds of problem need more number of distance equations than number of variables .

1.5 Methodology of Research

In this paper we describe a closed-form solution to determine the coordinates of the underwater sensors having only one beacon node at the surface and the beacon node are assumed to be in a plane, which is approximately parallel to the plan created by the three sensors. Here we assumed that the distance measurement between the beacon and sensors are possible, we described distance measurement in chapter 3. For simplicity we assume that the submerged sensors are stationary for a short period of time, during which the measurements of the distances from six different locations of the beacon are taken [6].

Chapter 2

Related Work

Among the first underwater acoustic systems was the submarine communication system developed in the USA around the end of the Second World War [7].

In [8], Duff and Muller proposed a method to solve the multilateration equations by means of nonlinear least square optimization when positions are not known. The algorithm is based on degree-of-freedom analysis – which says enough measurements from different positions will provide enough equations to solve the problem.

In [9], 3D Euclidean distance estimation method requires the need of a certain number of neighboring nodes to measure inter-node distances and where error is propagated through the system due to its recursive nature.

In[10], localization in UASNs faces some unavoidable challenges. Global positioning system (GPS) is commonly used in TWSNs, while it is impractical in UASNs since GPS signals cannot propagate in water. Radio signal propagates at long distances through water only at extra low frequency between 30 Hz and 300 Hz, which requires long antennae and high propagation power [11]. Moreover, the variable speed of sound and the node mobility caused by water current aggravate new challenges to localization issues in UASNs.

Due to the challenges mentioned above, localization algorithms in UASNs should be specifically designed. Experiments show that acoustic signal attenuates less and travels further in the water. Thus, acoustic communication is assumed to be the most promising mode for underwater communication. Both range-based algorithms and range-free algorithms have their own advantages and disadvantages. We will overview some representative range-based and range-free localization algorithms in Sections 2.1 and 2.2, respectively.

2.1. Range-Based Algorithms

Othman et al. proposed a node discovery and localization protocol (NDLP) for UASNs [12,13]; it is an anchor-free and range-based localization method. Localization begins with a node discovery phase by a seed node, which is aware of its self-position and selects other seeds iteratively. Seed nodes are responsible for assisting the other nodes with localization. Large scale of unknown nodes can be localized by selecting seed nodes continuously. However, the node discovery phase will spend much time and consume more energy, since each node participates in message exchange in order to select the seed nodes. What is more, in sparse sensor networks, it is possible that no sensor node can be selected as seed nodes, which will result in failure of localization.

Isik and Akan proposed a localization algorithm called three-dimensional underwater localization (3DUL) [14]. 3DUL algorithm can achieve network wide robust 3D localization by using a distributed and iterative algorithm. It is emphasized that 3DUL algorithm exploits only three surface buoys for localization initially. 3DUL algorithm requires that sensor nodes in the network should be equipped with conductivity, temperature, and depth (CTD) sensors to estimate the sound speed. The depth information is also used for the projection of the anchor nodes. Generally speaking, 3DUL contains two phases. During the first phase, unknown nodes estimate their distances to neighboring anchor nodes. In the second phase, unknown nodes use these pair wise distances and depth information to project the anchor nodes onto their horizontal levels and form virtual geometric structures. If the structure is robust, the unknown node locates itself by using dynamic trilateration method and upgrades to be an anchor node to assist other unknown nodes in localization. This process dynamically repeats to localize as many nodes as possible. Simulations show that the 3DUL algorithm can achieve relatively high accuracy in UASNs.

2.2. Range-Free Algorithms

Chandrasekhar and Seah hold the view that, for large-scale UASNs, obtaining the exact location of each unknown node may be infeasible [15]. Therefore, they proposed a range-free algorithm called area localization scheme (ALS) to achieve localization. It is notable that ALS estimates the position of each unknown node within a certain area rather than its exact location. In ALS, anchor nodes are responsible for sending out signals under different levels of power to localize unknown nodes. Unknown nodes simply listen to the signals and record the anchor nodes' IDs and their corresponding power levels. Then the collected data and the recorded information are sent to the sink node, which is able to draw out the map of areas divided by all the anchor nodes' transmitting signals and localize the unknown nodes. ALS is a range-free localization algorithm without synchronization requirement. However, since ALS is a coarse localization scheme, it is not suitable for accurate localization and instant information gathering.

Ma and Hu proposed a maximum-likelihood source localization approach (MLSL) for UASNs [16]. Sensor arrays are used during localization and each sensor node is equipped with an array. Sensor nodes are attached to the sensor arrays via wire connections. Each target which waits to be localized periodically emits a narrow-band acoustic signal. Sensor nodes which have received the signal can obtain the target locations and signal amplitudes by using the negative log-likelihood function. The maximum likelihood estimation of the target location is obtained based on the global likelihood function, which is the sum of the local likelihood function. MLSL approach does not require distance measurement and time synchronization. It is analyzed that computation overhead of sensor nodes and targets is low, while communication overhead and energy consumption are high, as all the local likelihood functions are forwarded to a fusion center. MLSL is not suitable for large-scale UASNs due to the fact that sensor nodes are attached to the sensor arrays via wire connections and global wireless network architecture.

Chapter 3

Solvability in UWSN localization systems

3.1. Problem Field

In our proposed model we assumes at least three submerged sensors deployed in the water and a floating beacon ,also measuring the multiple distances between the beacon and sensors, those locations of the beacon are assumed to be in a plane, which is approximately parallel to the plan created by the three sensors. A solvable configuration of one beacon with three submerged sensors is denoted in Fig.3.1.

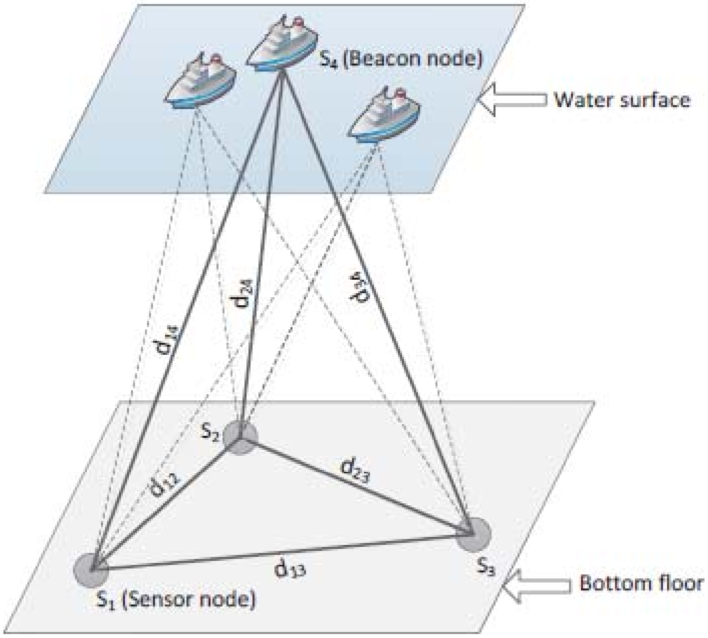

Fig. 3.1. Subset composed of one beacon and three submerged sensors

The measurements of the distances from six different locations of the beacon are taken , for simplicity we assume that the submerged sensors are stationary for a short period of time. Our proposed mechanism exploits the advantage of both radio and acoustic signal propagation in sea water .Since most of the marine explorations take place in shallow water, our proposed model has wide ranging practical applications.

3.2 Environmental limitation:

While node deployment in terrestrial environment is relatively straightforward and the corresponding deployment in underwater environment is more adverse[2].So underwater environment despite those limitations, it poses some merits that could be exploited in determining coordinates. The region of interest on the ground is more likely occupied with buildings and trees which are the major factors for multipath propagation. Regarding signal propagation in water, acoustic signal propagates much further compare to radio signal; however, the speed of the acoustic signal is much slower than that of radio signal[6]. Multipath models as well as actual measurements taken from sea trials show that the energy of the direct path of the channel’s impulse response is not always the strongest. As a result, multipath signals can be mistaken for the direct signal and may significantly impact the accuracy of distance estimation[2]. There are some limitations and some typical measurements for radio and acoustic signals shows Table 3.1[6].

		Velocity	**Range**
Radio Signal	**Vacuum**	**3×10^8 m/s**	--
	Water	**$\approx2.25\times10^6$ m/s**	**1.8-323 m**
Acoustic Signal	**Vacuum**	**≈1500 m/s**	**--**
	Water	**V_A**	**1 - 100 km**

Table 3.1: Properties of radio and acoustic signals

The main environmental variable that we assume in our method to determine distances is the speed of acoustic signals in water[6]. Unlike the speed of light which is constant, the speed of sound underwater varies with water temperature, pressure and salinity, giving rise to refraction. Without measuring the sound speed, the accuracy of distance measurements based on time-of-arrival approaches may be degraded[2]. How the speed of acoustic will vary because of aforesaid factors is not considered in this study, but our mathematical model assumes it as a variable V_A .

3.3 Distance Measurement

Though in under water both radio and acoustic signal propagation have some limitations in different aspect , we will exploit each of their merits in our proposed method . Differential speed between radio and acoustic signals will be used to calculate the distance, while acoustic signal will be used for communication purposes. It increase the accuracy of the distance measurements. Even though the speed of radio signal is slightly less than that of in the vacuum, considering the problem domain, the speed variation will not have significant impact on the proposed localization method. On the other hand the speed of acoustic signal, which varies due to different environmental factors, is the main variable that we need to use for determine coordinate[6].

Our proposed model typically comprises the following steps:

Step-1:

The beacon can generate radio and acoustic signals at a same time. while measuring the internodes distances the environmental factors that affect the acoustic signal will be considered. Each sensor node will have a unique ID. Sensor nodes are stationary during the short measurement period[6].

Step-2:

Simultaneous generation of radio and acoustic signals by the beacon $S_j, j = 4,5....$ at t_0 .For any submerged sensors $S_i, i = 1,2,3$

(i)Sensors receive the radio signal immediately at

$$t_{R(rec)} \approx t_0 + \varepsilon$$

(ii)Sensor receives the acoustic signal after a while at $t_{A(rec)}$; here $t_{A(rec)} >> t_{R(rec)}$ due to speed of radio signal.

Time of acoustic signal travelled from beacon to sensors:

$$T_{ij(Travel), i=1,2,3; j=4,5,6.....} = t_{A(rec)} - t_{A(tra)}$$
$$= t_{A(rec)} - t_{R(tra)} \quad \because t_{A(tra)} = t_{R(tra)}$$

$$\therefore T_{ij(Travel)} \approx t_{A(rec)} - t_{R(rec)} \quad \because t_{R(rec)} \approx t_0 + \varepsilon$$

Sensor nodes send back the time $T_{ij(Travel)}$ with individual sensor's ID back to the beacon using acoustic signal.

Beacon node computes the distance between the beacon and sensors $d_{ij} = v_A \times T_{ij(travel)}$

3.4 Coordinates Computation

Only measurements available here to compute is the distance and typically it is considered as optimization problem where objective functions to be minimized have residuals of the distance equations. The variables of any localization problem are the 3D coordinates of the nodes. In principle more number of distance equations are needed than number of variables to solve this kind of problem. However, this approach known as degree of freedom analysis may not guarantee the unique solution in a nonlinear system. Trilateration or multilateration techniques that are nonlinear system usually used to determine the location or coordinates of the sensors in partial or full. According to Guevara et al. [17] methods depend heavily on initial conditions used and they circumvent the convergence problem by linearizing the trilateration equations.

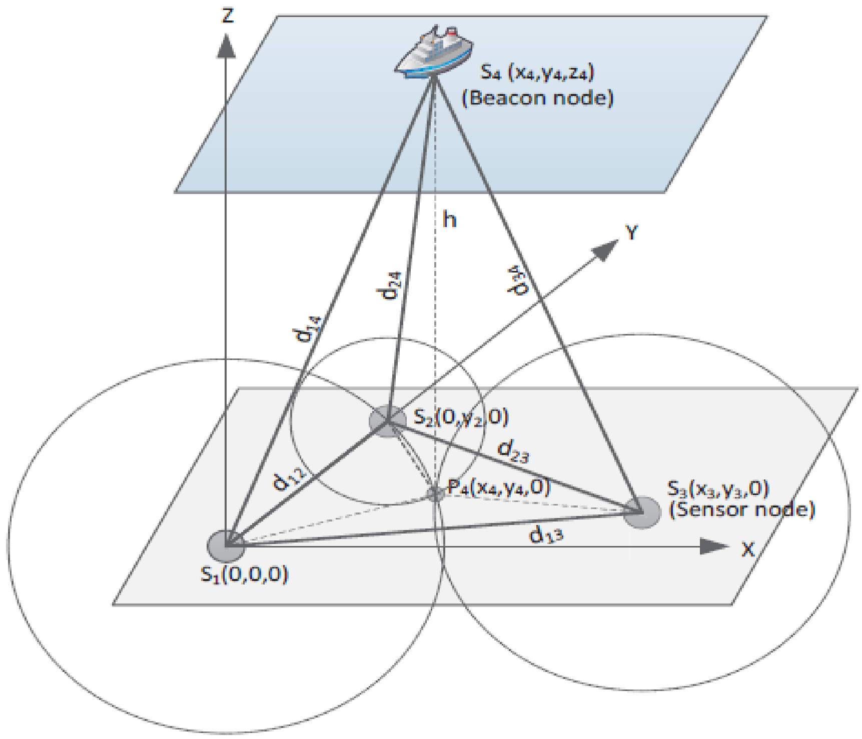

Fig.3.2. Coordinates determinations

In Fig-3.2. $S_j, j = 4,5....9$ is the initial subset composed of the beacon node and three sensor nodes $S_i, i = 1,2,3$. Here the sensor nodes $S_i, i = 1,2,3$ in the origin $(0,0,0)$ of the coordinate system. The distance between beacon and sensors $d_{14}, d_{24,} d_{34}....$ which are measured data and inter-sensor distances $d_{12}, d_{13,} d_{23}$ and V_t the volume of tetrahedron. The distance between beacon and sensors are measured data, and the volume of tetrahedron V_t formed by the beacon and sensors, which are unknown. By LPS configuration we need to write equation that will includes all known and unknown distances.

By grouping known-unknown variables of (1), we get

$$d_{34}^{2}\left(d_{12}^{2}-d_{23}^{2}-d_{13}^{2}\right)+d_{14}^{2}\left(\tfrac{d_{23}^{4}}{d_{12}^{2}}-d_{23}^{2}-\tfrac{d_{13}^{2}d_{23}^{2}}{d_{12}^{2}}\right)+d_{24}^{2}\left(\tfrac{d_{13}^{4}}{d_{12}^{2}}-\tfrac{d_{13}^{2}d_{23}^{2}}{d_{12}^{2}}-d_{13}^{2}\right)$$
$$-\left(d_{14}^{2}d_{24}^{2}+d_{14}^{2}d_{34}^{2}-d_{24}^{2}d_{34}^{2}-d_{14}^{4}\right)\tfrac{d_{23}^{2}}{d_{12}^{2}}-\left(d_{34}^{2}d_{24}^{2}-d_{14}^{2}d_{34}^{2}+d_{14}^{2}d_{24}^{2}-d_{24}^{4}\right)$$
$$\tfrac{d_{13}^{2}}{d_{12}^{2}}+\left(144\tfrac{V_t^2}{d_{12}^{2}}+d_{13}^{2}d_{23}^{2}\right)=\left(d_{24}^{2}d_{34}^{2}-d_{34}^{4}+d_{14}^{2}d_{34}^{2}-d_{14}^{2}d_{24}^{2}\right)$$

Here, the unknown terms are $\left(d_{12}^{2}-d_{23}^{2}-d_{13}^{2}\right)$, $\left(\tfrac{d_{23}^{4}}{d_{12}^{2}}-d_{23}^{2}-\tfrac{d_{13}^{2}d_{23}^{2}}{d_{12}^{2}}\right)$, $\left(\tfrac{d_{13}^{4}}{d_{12}^{2}}-\tfrac{d_{13}^{2}d_{23}^{2}}{d_{12}^{2}}-d_{13}^{2}\right)$, $\tfrac{d_{23}^{2}}{d_{12}^{2}}$, $\tfrac{d_{13}^{2}}{d_{12}^{2}}$ and $\left(144\tfrac{V_t^2}{d_{12}^{2}}+d_{13}^{2}d_{23}^{2}\right)$

We define the new unknown variables X_n of the linearized equation, where each variable corresponds with the n^th element of matrix X defined

$$d_{14}^{2}x_1+d_{24}^{2}x_2+d_{34}^{2}x_3-\left(d_{14}^{2}-d_{34}^{2}\right)\left(d_{24}^{2}-d_{14}^{2}\right)x_4$$
$$-\left(d_{24}^{2}-d_{14}^{2}\right)\left(d_{34}^{2}-d_{24}^{2}\right)x_5+x_6=\left(d_{24}^{2}-d_{34}^{2}\right)\left(d_{34}^{2}-d_{14}^{2}\right)\quad\text{..(1)}$$

Based on the local positioning system configuration of Fig.3.2,we need to write equations that will include all known and unknown distances. For that matter, we express the volume of tetrahedron v_t using Cayley-Menger determinant as following:

$$288V_t^2=\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & d_{12}^2 & d_{13}^2 & d_{14}^2 \\ 1 & d_{12}^2 & 0 & d_{23}^2 & d_{24}^2 \\ 1 & d_{13}^2 & d_{23}^2 & 0 & d_{34}^2 \\ 1 & d_{14}^2 & d_{24}^2 & d_{34}^2 & 0 \end{vmatrix}\quad\text{..(2)}$$

which can be write resembles linear form as follow :

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b_1$$

As we have six unknown we need at least six measurements, which could be done following the same procedure the beacon node $s_j, j = 4,5....9$ in six different places and measuring the distances in the vicinity of s_4.

Eventually we get m linear equations of the form, That could be expressed in $Ax = b$ linear form.

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1,\\
&a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)\\
&\vdots\\
&a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m,
\end{aligned}$$

Here,

$$X = \begin{bmatrix} \left(\frac{d_{23}^4}{d_{12}^2} - d_{23}^2 - \frac{d_{13}^2 d_{23}^2}{d_{12}^2}\right) \\ \left(\frac{d_{13}^4}{d_{12}^2} - \frac{d_{13}^2 d_{23}^2}{d_{12}^2} - d_{13}^2\right) \\ \left(d_{12}^2 - d_{23}^2 - d_{13}^2\right) \\ \frac{d_{23}^2}{d_{12}^2} \\ \frac{d_{13}^2}{d_{12}^2} \\ \left(144 \frac{V_t^2}{d_{12}^2} + d_{13}^2 d_{23}^2\right) \end{bmatrix}$$

$$b = \begin{bmatrix} \left(d_{24}^2 - d_{34}^2\right)\left(d_{34}^2 - d_{14}^2\right) \\ \left(d_{25}^2 - d_{35}^2\right)\left(d_{35}^2 - d_{15}^2\right) \\ \vdots \\ \left(d_{29}^2 - d_{39}^2\right)\left(d_{39}^2 - d_{19}^2\right) \end{bmatrix}$$

From the linear representation, after finding X_1, x_2, x_3, x_4, x_5 and x_6 we calculate the unknowns distance of d_{12}, d_{13} and d_{23} as following:

$$d_{12}^2 = \frac{x_3}{(1 - x_4 - x_5)}$$

$$d_{13}^2 = \frac{x_3 x_5}{(1 - x_4 - x_5)}$$

$$d_{23}^2 = \frac{x_3 x_4}{(1 - x_4 - x_5)}$$

The sensors s_1, $(0,0,0)$, s_2 $(0, y_2, 0)$, s_3 are $(x_3, y_3, 0)$ the inter-sensors distances could be written with respect to coordinates of the sensors as $d_{12}^2 = y_2^2$, $d_{13}^2 = x_3^2 + y_3^2$, $d_{23}^2 = x_3^2 + (y_3 - y_2)^2$. the above values we get the unknown variables d_{12}, d_{13} and d_{23} are known computed distances.

$$y_2 = d_{12}, \; y_3 = \frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}, \; x_3 = \sqrt{\left(d_{13}^2 - \left(\frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}\right)^2\right)}$$

Chapter 4

Experimental Results and Analysis

A simulation of the proposed method to determinate coordinates of submerged sensors as described in chapter 3 was performed to verify the method. The experiment was designed based on 3-D space. As the distance measurement between bacon and sensors is possible so a group of three sensors are placed at (0, 0, 0), (0, 60, 0) and (85, 90, 0) and a floating bacon randomly moved in plane which is parallel to the XY plane where the sensors are in 3D- space. The coordinates of the sensors are randomly chosen. Z-coordinates of sensors is always kept zero to satisfy that all sensors are situated in same plane and for computational simplicity one of the coordinates are placed at the origin. This proposed method has been simulated using Matlab(v-2014b). To simulate the proposed method 50 datasets were taken. Each datasets contains six different positions of the beacon to get distance measurement and it has been randomly moved around to six different coordinates within close proximity. Gaussian noise has been added While calculating the true euclidian distance from six different bacon nodes to sensors S_1, S_2 and S_3.To calculate the coordinates of sensors we need the inner distances between sensors S_1, S_2 and S_3.

After solve the linear equation (3) which is formed by Cayley-Menger Determinent(2) we find the inner disteances between sensors S_1, S_2 and S_3. These distances may contain some gaussian error because of it calculated form the Gasussian errored distances between bacon and sensors. The distance errors among the sensors are given below :

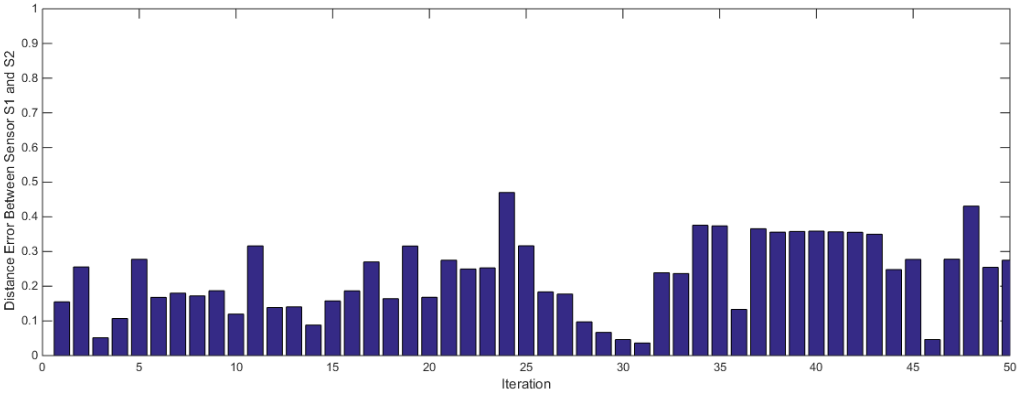

Fig.4.3.Distance error between sensor S_1 and S_2

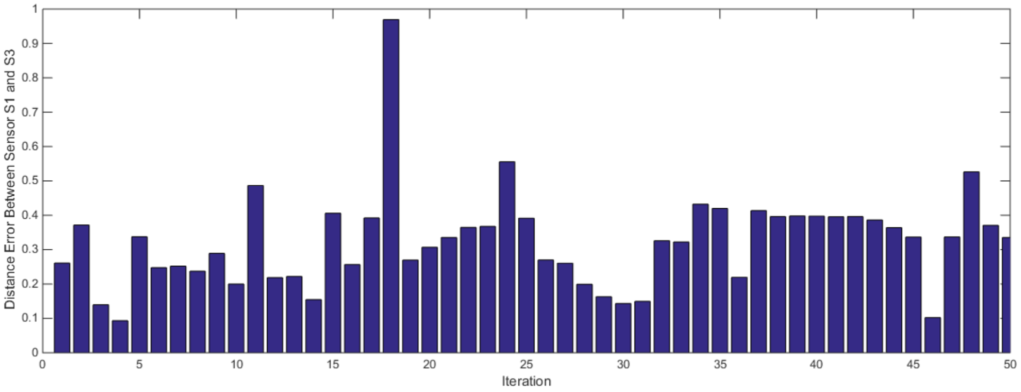

Fig.4.4.Distance error between sensor S_1 and S_3

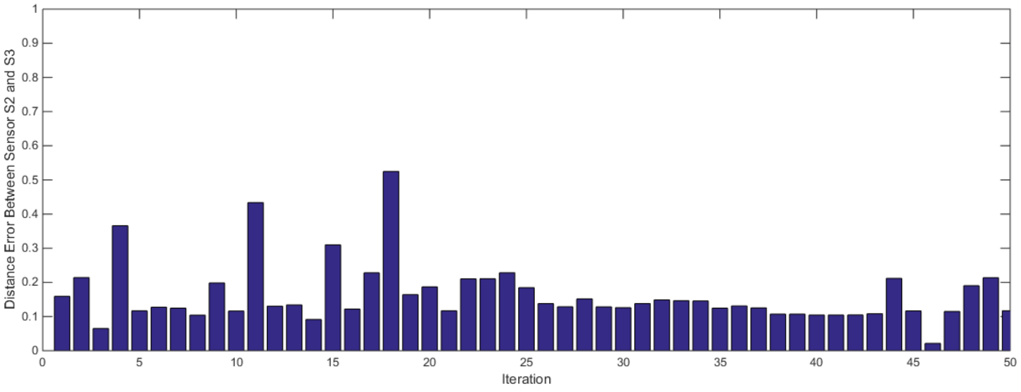

Fig.4.5.Distance error between sensor S_2 and S_3

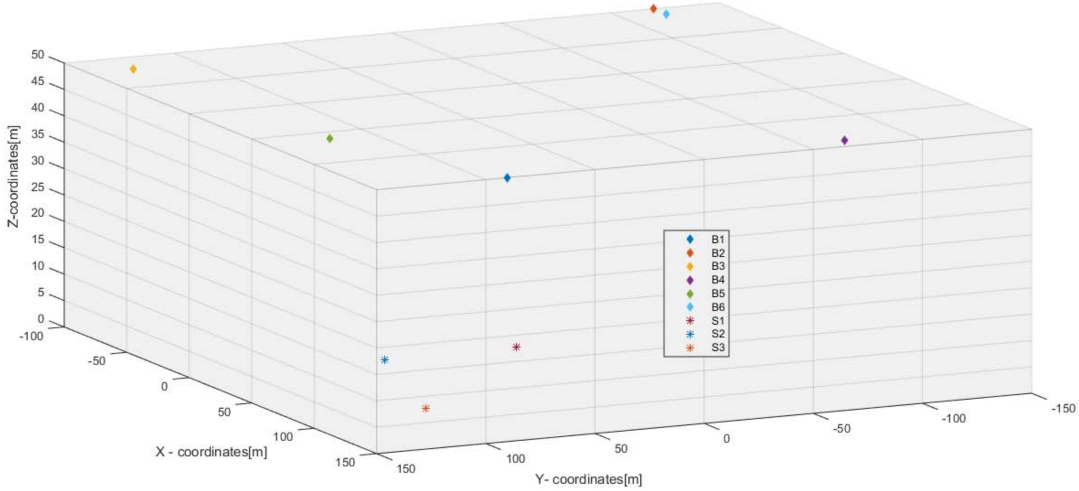

Fig.4.6.Calculated sensors positions with proposed method

Errors in coordinates for sensors S_2 and S_3 are shown in Fig.4.7 and Fig.4.8 respectively for 50 datasets. It should be noted that sensor S_1 is placed at the reference coordinate (0,0,0); hence producing no error in coordinate determination for S_1 , moreover S_2 and S_3 are computed with respect to S_1 .

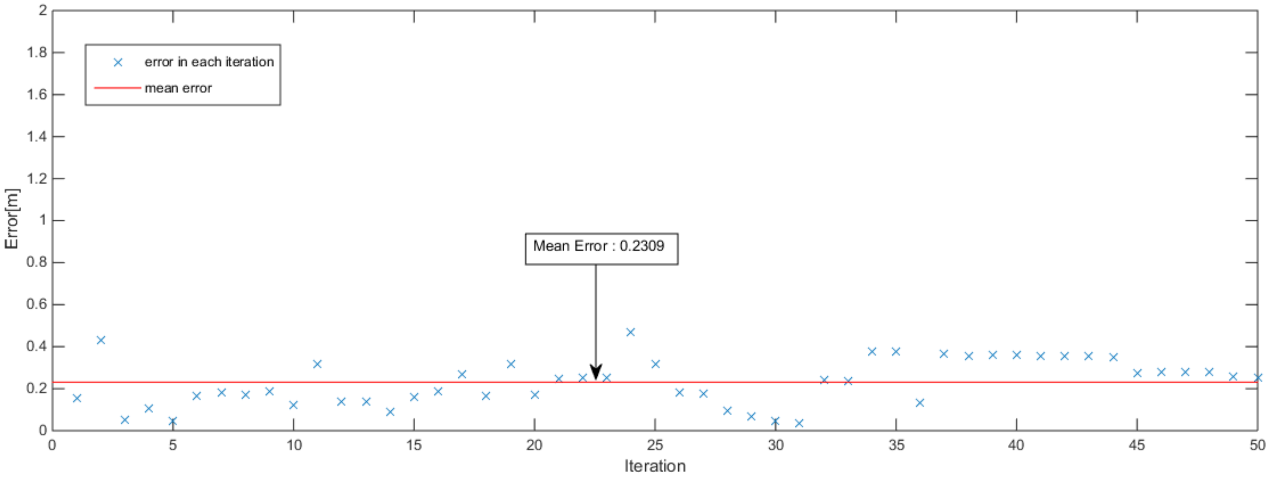

Fig.4.7.Mean error for S_2

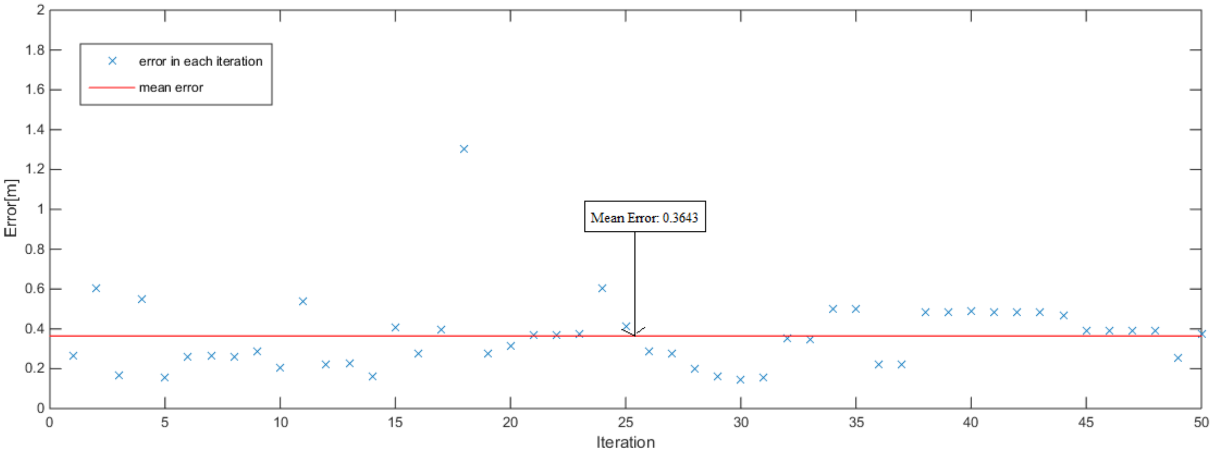

Fig.4.8.Mean error for S_3

Table 4.2 compares the positional error of sensor S_2 and S_3 when distances between the beacon and sensors are true Euclidean as well as with Gaussian noise. Positional error generates with true Euclidean is almost negligible which validates the proposed mathematical model. Besides, error with Gaussian noise is within acceptable range considering 150m water column and usual physical sizes of the sensors. It also shows that the mean error is 0.2309m and 0.3643m with standard deviation of 0.1104 and 0.1852 respectively.

Sensors (Unknown coordinates)	Mean error(m) (with Gaussian Noise)	Standard deviation of error distribution
S2	0.2309	0.1104
S3	0.3643	0.1852

Table 4.2: Generation of Errors

Chapter 5

Conclusion and Future Work

We have described a measurement techniques in under water sensor localization by only one beacon node. We use Multilateration technique to determine the location of the sensors with respect to beacon nodes where distance between them is measured considering the acoustic and radio signal. Moreover Cayley–Menger determinant is used to determined the nodes coordinates ,it's reduce the impact of distance measurement error on the location estimation. Simulation result validate the mathematical model by computing coordinates of sensors with negligible error. Therefore the coordinates are acceptable error range after adding Gaussian noise into the distance measurement.

Our future plan to determined the coordinates of underwater deployed sensors nodes with respect to one beacon node .where the position of sensors node are non parallel plan to beacon node .

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Appendix

Data Set

S_1	Bacon Nodes (from six different position)	Sensor Nodes				Sensor Nodes with Gaussian Error				Distance Error From Original Position
			X	Y	Z		X	Y	Z	
1	B1 = [150 90 50] B2 = [-100 -120 50] B3 = [-80 130 50] B4 = [140 -70 50] B5 = [60 120 50] B6 = [-90 -130 50]	S_1	0	0	0	S_1	0	0	0	0
		S_2	0	60	0	S_2	0	60.1550	0	0.1550
		S_3	85	90	0	S_3	85.1453	90.2218	0	0.2651
2	B1 = [50 50 50]; B2 = [-70 -120 50]; B3 = [-80 110 50]; B4 = [90 -130 50]; B5 = [60 100 50]; B6 = [-100 -120 50];					S_1	0	0	0	0
						S_2	0	60.4310	0	0.4310
						S_3	85.1480	90.5835	0	0.6020
3	B1 = [100 90 50]; B2 = [-100 -120 50]; B3 = [-80 130 50]; B4 = [140 -70 50]; B5 = [60 110 50]; B6 = [-90 -130 50];					S_1	0	0	0	0
						S_2	0	60.0512	0	0.05129
						S_3	85.0285	90.1651	0	0.1676
4	B1 = [100 90 50]; B2 = [-110 -120 50]; B3 = [-80 130 50]; B4 = [140 -70 50]; B5 = [60 110 50]; B6 = [-90 -130 50];					S_1	0	0	0	0
						S_2	0	60.1070	0	0.1070
						S_3	84.5421	90.3025	0	0.5488
5	B1 = [100 90 50]; B2 = [-110 -120 50]; B3 = [-80 130 50]; B4 = [140 -80 50]; B5 = [60 110 50]; B6 = [-100 -130 50];					S_1	0	0	0	0
						S_2	0	60.0462	0	0.04621
						S_3	84.9840	90.1556	0	0.1564
6	B1 = [100 90 50]; B2 = [-110 -120 50]; B3 = [-80 130 50]; B4 = [140 -80 50];					S_1	0	0	0	0
						S_2	0	60.1679	0	0.1679
						S_3	85.1109	90.2359	0	0.2607

	B5 = [160 -110 50]; B6 = [-100 -130 50];						
7	B1 = [100 90 50]; B2 = [-110 -120 50]; B3 = [-80 130 50]; B4 = [40 -80 50]; B5 = [160 -110 50]; B6 = [-90 -130 50];		S_1	0	0	0	0
			S_2	0	60.1801	0	0.1801
			S_3	85.1102 868	90.2424	0	0.2663
8	B1 = [100 90 50]; B2 = [-110 -120 50]; B3 = [-180 130 50]; B4 = [140 -80 50]; B5 = [160 -110 50]; B6 = [-90 -130 50];		S_1	0	0	0	0
			S_2	0	60.1721	0	0.1721
			S_3	85.0841	90.2468		0.26075
9	B1 = [100 90 50]; B2 = [-119 -120 50]; B3 = [-180 130 50]; B4 = [140 -145 50]; B5 = [160 -110 50]; B6 = [-90 -130 50];		S_1	0	0	0	0
			S_2	0	60.1871	0	0.1871
			S_3	85.2042	90.2048	0	0.2893
10	B1 = [100 90 50]; B2 = [-119 -120 50]; B3 = [-180 156 50]; B4 = [140 -145 50]; B5 = [160 -115 50]; B6 = [-90 -170 50];		S_1	0	0	0	0
			S_2	0	60.1195	0	0.1195
			S_3	85.1026	90.1779	0	0.20544
11	B1 = [102 90 50]; B2 = [-119 -120 50]; B3 = [-180 156 50]; B4 = [140 -148 50]; B5 = [167 -115 50]; B6 = [90 -170 50];		S_1	0	0	0	0
			S_2	0	60.3162		0.3162
			S_3	85.5024	90.1943		0.5387
12	B1 = [102 111 50]; B2 = [-119 -122 50]; B3 = [-180 136 50]; B4 = [-140 -148 50]; B5 = [167 -115 50]; B6 = [90 -170 50];		S_1	0	0	0	0
			S_2	0	60.1386	0	0.1386
			S_3	85.1217	90.1858		0.2221

13	B1 = [102 111 50]; B2 = [-111 -122 50]; B3 = [-188 133 50]; B4 = [-144 -144 50]; B5 = [166 -115 50]; B6 = [99 -177 50];		S_1	0	0	0	0
			S_2	0	60.1403		0.14035
			S_3	85.1257	90.1865		0.2249
14	B1 = [102 161 50]; B2 = [-111 -122 50]; B3 = [208 -119 50]; B4 = [-104 -167 50]; B5 = [105 -195 50]; B6 = [199 -197 50];		S_1	0	0	0	0
			S_2	0	60.0882		0.08823
			S_3	85.0798	90.1372		0.1587
15	B1 = [-102 161 50];		S_1	0	0	0	0

	B2 = [111 -122 50]; B3 = [89 -119 50]; B4 = [194 -137 50]; B5 = [105 -165 50]; B6 = [199 -117 50];		S_2	0	60.1578	0	0.1578
			S_3	85.2809	90.2933	0	0.40617
16	B1 = [-55 71 50]; B2 = [101 -102 50]; B3 = [89 -100 50]; B4 = [94 -130 50]; B5 = [-105 -165 50]; B6 = [98 115 50];		S_1	0	0	0	0
			S_2	0	60.1866	0	0.1866
			S_3	85.1060	90.2527	0	0.2740
17	B1 = [-55 50 50]; B2 = [-101 -102 50]; B3 = [87 -100 50]; B4 = [90 -130 50]; B5 = [-105 -165 50]; B6 = [-98 -115 50];		S_1	0	0	0	0
			S_2	0	60.2700	0	0.2700
			S_3	85.2203	90.3314	0	0.3980
18	B1 = [55 60 50]; B2 = [101 102 50]; B3 = [87 -100 50]; B4 = [90 -130 50]; B5 = [85 165 50]; B6 = [98 -195 50];		S_1	0	0	0	0
			S_2	0	59.8356	0	0.1643
			S_3	85.0275	91.3029	0	1.3032
19	B1 = [55 -60 50]; B2 = [-101 102 50]; B3 = [-87 -100 50]; B4 = [-92 -130 50]; B5 = [-93 165 50]; B6 = [-102 195 50];		S_1	0	0	0	0
			S_2	0	59.6839	0	0.3160
			S_3	84.7676	89.8483	0	0.27750
20	B1 = [55 -56 50]; B2 = [-101 102 50]; B3 = [-87 -88 50]; B4 = [-92 -93 50]; B5 = [-164 165 50]; B6 = [-102 102 50];		S_1	0	0	0	0
			S_2	0	60.1683	0	0.16832
			S_3	85.1631	90.2682	0	0.3140
21	B1 = [86 75 50]; B2 = [-92 -112 50]; B3 =[-85 -108 50]; B4 = [95 -93 50]; B5 = [60 150 50]; B6 = [-125 125 50];		S_1	0	0	0	0
			S_2	0	60.2481	0	0.2480
			S_3	85.2030	90.3091	0	0.3697
22	B1 = [86 75 50]; B2 = [-92 -112 50]; B3 = [-85 -108 50]; B4 = [95 -93 50]; B5 = [61 150 50]; B6 = [-125 125 50];		S_1	0	0	0	0
			S_2	0	60.2496	0	0.2496
			S_3	85.2014	90.3114	0	0.3709
23	B1 = [86 75 50]; B2 = [-92 -112 50]; B3 = [-81 -108 50]; B4 = [95 -93 50];		S_1	0	0	0	0
			S_2	0	60.2529518059985	0	0.2529

	B5 = [61 155 50]; B6 = [-125 125 50];		S_3	85.2015825409506	90.3155363965403	0	0.3744
24	B1 = [40 75 50]; B2 = [-92 -112 50]; B3 = [-81 -108 50]; B4 = [55 -90 50]; B5 = [61 155 50]; B6 = [-125 125 50];		S_1	0	0	0	0
			S_2	0	60.4703796194605	0	0.4703
			S_3	85.2078221299772	90.5676791300263	0	0.6045
25	B1 = [40 75 50]; B2 = [-30 -112 50]; B3 = [-81 -108 50]; B4 = [55 -90 50]; B5 = [61 155 50]; B6 = [-125 125 50];		S_1	0	0	0	0
			S_2	0	60.3167147660634	0	0.3167
			S_3	85.1760523017978	90.3721880407260	0	0.4117
26	B1 = [80 -67 50]; B2 = [-100 -106 50]; B3 = [-105 -100 50]; B4 = [97 -96 50]; B5 = [130 169 50]; B6 = [-120 126 50];		S_1	0	0	0	0
			S_2	0	60.1836	0	0.1837
			S_3	85.1204	90.2575	0	0.2843
27	B1 = [85 -77 50]; B2 = [-100 -106 50]; B3 = [-105 -100 50]; B4 = [97 -96 50]; B5 = [130 160 50]; B6 = [-120 126 50];		S_1	0	0	0	0
			S_2	0	60.1775	0	0.1775
			S_3	85.1091	90.2551	0	0.2776
28	B1 = [85 -77 50]; B2 = [-105 -106 50]; B3 = [-105 -100 50]; B4 = [97 -96 50]; B5 = [110 160 50]; B6 = [-120 126 50];		S_1	0	0	0	0
			S_2	0	60.0974	0	0.0974
			S_3	85.1481	90.1341	0	0.1998
29	B1 = [85 -77 50]; B2 = [-105 -106 50]; B3 = [-105 -100 50]; B4 = [100 -96 50]; B5 = [110 160 50]; B6 = [-150 120 50];		S_1	0	0	0	0
			S_2	0	60.0670	0	0.0670
			S_3	85.1211	90.1099	0	0.1636
30	B1 = [85 -75 50]; B2 = [-105 -110 50]; B3 = [-105 -100 50]; B4 = [100 -96 50]; B5 = [110 160 50]; B6 = [-150 120 50];		S_1	0	0	0	0
			S_2	0	60.0461	0	0.04614
			S_3	85.1204271414	90.08342133792	0	0.1464
31	B1 = [85 -75 50]; B2 = [-110 -110 50]; B3 = [-105 -100 50]; B4 = [140 -96 50];		S_1	0	0	0	0
			S_2	0	60.0362	0	0.03624
			S_3	85.1297	90.0834	0	0.1542

	B5 = [110 160 50]; B6 = [-150 120 50];						
32	B1 = [50 -75 50]; B2 = [-72 -110 50]; B3 = [-105 -100 50]; B4 = [90 -96 50]; B5 = [110 160 50]; B6 = [-150 120 50];		S_1	0	0	0	0
			S_2	0	60.2388		0.2388
			S_3	85.1258	90.3293		0.3526
33	B1 = [50 -75 50]; B2 = [-72 -110 50]; B3 = [-105 -100 50]; B4 = [95 -96 50]; B5 = [110 160 50]; B6 = [-135 120 50];		S_1	0	0	0	0
			S_2	0	60.2366	0	0.2366
			S_3	85.1236	90.3265	0	0.3491
34	B1 = [50 75 50]; B2 = [-72 -110 50]; B3 = [-105 -110 50]; B4 = [95 -96 50]; B5 = [110 160 50]; B6 = [-135 120 50];		S_1	0	0	0	0
			S_2	0	60.3760	0	0.3760
			S_3	85.1162	90.4844	0	0.4982
35	B1 = [55 75 50]; B2 = [-72 -110 50]; B3 = [-105 -110 50]; B4 = [95 -96 50]; B5 = [110 150 50]; B6 = [-135 120 50];		S_1	0	0	0	0
			S_2	0	60.3741	0	0.3741
			S_3	85.0906	90.4919	0	0.5002
36	B1 = [102 161 50]; B2 = [-111 -129 50]; B3 = [108 -119 50]; B4 = [-104 -154 50]; B5 = [106 -195 50]; B6 = [179 -197 50];		S_1	0	0	0	0
			S_2	0	60.1331	0	0.1331
			S_3	85.1194	90.1892	0	0.2237
37	B1 = [57 75 50]; B2 = [-77 -110 50]; B3 = [-115 -110 50]; B4 = [95 -96 50]; B5 = [110 150 50]; B6 = [-135 120 50];		S_1	0	0	0	0
			S_2	0	60.3658	0	0.3658
			S_3	85.0919	90.4819	0	0.4906
38	B1 = [57 75 50]; B2 = [-77 -110 50]; B3 = [-115 -110 50]; B4 = [95 -96 50]; B5 = [115 150 50]; B6 = [-155 125 50];		S_1	0	0	0	0
			S_2	0	60.35597526272	0	0.35595
			S_3	85.0706	90.4779	0	0.4831
39	B1 = [57 75 50]; B2 = [-70 -110 50]; B3 = [-116 -110 50]; B4 = [96 -96 50]; B5 = [115 150 50]; B6 = [-155 125 50];		S_1	0	0	0	0
			S_2	0	60.3578	0	0.3578
			S_3	85.0705	90.4805	0	0.4856
40	B1 = [57 75 50]; B2 = [-70 -110 50]; B3 = [-116 -110 50];		S_1	0	0	0	0
			S_2	0	60.3589	0	0.3589
			S_3	85.0670	90.4830		0.4876

	B4 = [90 -96 50]; B5 = [117 150 50]; B6 = [-135 125 50];						
41	B1 = [57 75 50]; B2 = [-77 -110 50]; B3 = [-116 -110 50]; B4 = [92 -96 50]; B5 = [117 150 50]; B6 = [-135 125 50];		S_1	0	0	0	0
			S_2	0	60.3568	0	0.3568
			S_3	85.0674	90.4802	0	0.4849
42	B1 = [57 75 50]; B2 = [-78 -110 50]; B3 = [-118 -110 50]; B4 = [93 -96 50]; B5 = [117 150 50]; B6 = [-135 125 50];		S_1	0	0	0	0
			S_2	0	60.3559	0	0.3559
			S_3	85.0676	90.4790	0	0.4837
43	B1 = [57 75 50]; B2 = [-78 -110 50]; B3 = [-118 -110 50]; B4 = [94 -96 50]; B5 = [115 150 50]; B6 = [-127 125 50];		S_1	0	0	0	0
			S_2	0	60.3556	0	0.3556
			S_3	85.0717	90.4771	0	0.4824
44	B1 = [57 75 50]; B2 = [-78 -110 50]; B3 = [-118 -110 50]; B4 = [94 -92 50]; B5 = [115 150 50]; B6 = [-127 125 50];		S_1	0	0	0	0
			S_2	0	60.3500	0	0.3500
			S_3	85.0706	90.4643	0	0.4696
45	B1 = [86 75 50]; B2 = [-90 -110 50]; B3 = [-118 -110 50]; B4 = [95 -92 50]; B5 = [115 150 50]; B6 = [-127 125 50];		S_1	0	0	0	0
			S_2	0	60.2751	0	0.2751
			S_3	85.0877	90.3780	0	0.3880
46	B1 = [86 75 50]; B2 = [-92 -110 50]; B3 = [-108 -110 50]; B4 = [95 -92 50]; B5 = [115 150 50]; B6 = [-127 125 50];		S_1	0	0	0	0
			S_2	0	60.2774	0	0.2774
			S_3	85.0870	90.3809	0	0.3907
47	B1 = [86 75 50]; B2 = [-92 -110 50]; B3 = [-108 -110 50]; B4 = [95 -93 50]; B5 = [115 150 50]; B6 = [-127 125 50];		S_1	0	0	0	0
			S_2	0	60.2778	0	0.27782
			S_3	85.0872	90.3819	0	0.3918
48	B1 = [86 75 50]; B2 = [-92 -110 50]; B3 = [-108 -110 50]; B4 = [95 -93 50]; B5 = [116 150 50]; B6 = [-125 125 50];		S_1	0	0	0	0
			S_2	0	60.2781	0	0.2781
			S_3	85.0850	90.3830	0	0.3923
49	B1 = [86 75 50];		S_1	0	0	0	0

	B2 = [-92 -110 50]; B3 = [-82 -110 50]; B4 = [95 -93 50]; B5 = [60 150 50]; B6 = [-125 125 50];		S_2	0	60.2555	0	0.2555
			S_3	85.2051	90.3175	0	0.2555
50	B1 = [86 75 50]; B2 = [-92 -110 50]; B3 = [-85 -110 50]; B4 = [95 -93 50]; B5 = [60 150 50]; B6 = [-125 125 50];		S_1	0	0	0	0
			S_2	0	60.2546	0	0.2546
			S_3	85.2048	90.3164	0	0.3770

Code:

```
% Beacon Nodes

B1 = [ 150     90   50];
B2 = [-100 -120   50];
B3 = [  -80   130  50];
B4 = [ 140    -70   50];
B5 = [   60   120   50];
B6 = [ -90  -130   50];


% Sensors S1 , S2 , S3
S1 = [0 0 0]; % Sensor Coordinate
S2 = [0 60 0];
S3 = [85 90 0];

% from 1st Beacon Node
d14 = pdist2(B1,S1,'euclidean');
d24 = pdist2(B1,S2,'euclidean');
d34 = pdist2(B1,S3,'euclidean');

% from 2nd Beacon Node
d15 = pdist2(B2,S1,'euclidean');
d25 = pdist2(B2,S2,'euclidean');
d35 = pdist2(B2,S3,'euclidean');



% from 3rd Beacon Node
d16 = pdist2(B3,S1,'euclidean');
d26 = pdist2(B3,S2,'euclidean');
d36 = pdist2(B3,S3,'euclidean');

% from 4th Beacon Node
d17 = pdist2(B4,S1,'euclidean');
d27 = pdist2(B4,S2,'euclidean');
d37 = pdist2(B4,S3,'euclidean');

% from 5th Beacon Node
d18 = pdist2(B5,S1,'euclidean');
d28 = pdist2(B5,S2,'euclidean');
d38 = pdist2(B5,S3,'euclidean');

% from 6th Beacon Node
d19 = pdist2(B6,S1,'euclidean');
d29 = pdist2(B6,S2,'euclidean');
d39 = pdist2(B6,S3,'euclidean');
```

```
%%%%%%%%%%%%%%%% Adding Gaussian Error %%%%%%%%%%%%%%%%

% 1st Beacon Node
d14 = d14+erf(d14);
d24 = d24+erf(d24);
d34 = d34+erf(d34);

% 2nd Beacon Node
d15 = d15+erf(d15);
d25 = d25+erf(d25);
d35 = d35+erf(d35);

% 3rd Beacon Node
d16 = d16+erf(d16);
d26 = d26+erf(d26);
d36 = d36+erf(d36);

% 4th Beacon Node
d17 = d17+erf(d17);
d27 = d27+erf(d27);
d37 = d37+erf(d37);

% 5th Beacon Node
d18 = d18+erf(d18);
d28 = d28+erf(d28);
d38 = d38+erf(d38);

% 6th Beacon Node
d19 = d19+erf(d19);
d29 = d29+erf(d29);
d39 = d39+erf(d39);



% Cayley - Menger Determinant
a11=d14^2; a12=d24^2; a13=d34^2; a14=-(d14^2-d34^2)*(d24^2-d14^2); a15=-
(d24^2-d14^2)*(d34^2-d24^2); a16=1; b1=(d24^2-d34^2)*(d34^2-d14^2);
a21=d15^2; a22=d25^2; a23=d35^2; a24=-(d15^2-d35^2)*(d25^2-d15^2); a25=-
(d25^2-d15^2)*(d35^2-d25^2); a26=1; b2=(d25^2-d35^2)*(d35^2-d15^2);
a31=d16^2; a32=d26^2; a33=d36^2; a34=-(d16^2-d36^2)*(d26^2-d16^2); a35=-
(d26^2-d16^2)*(d36^2-d26^2); a36=1; b3=(d26^2-d36^2)*(d36^2-d16^2);
a41=d17^2; a42=d27^2; a43=d37^2; a44=-(d17^2-d37^2)*(d27^2-d17^2); a45=-
(d27^2-d17^2)*(d37^2-d27^2); a46=1; b4=(d27^2-d37^2)*(d37^2-d17^2);
a51=d18^2; a52=d28^2; a53=d38^2; a54=-(d18^2-d38^2)*(d28^2-d18^2); a55=-
(d28^2-d18^2)*(d38^2-d28^2); a56=1; b5=(d28^2-d38^2)*(d38^2-d18^2);
a61=d19^2; a62=d29^2; a63=d39^2; a64=-(d19^2-d39^2)*(d29^2-d19^2); a65=-
(d29^2-d19^2)*(d39^2-d29^2); a66=1; b6=(d29^2-d39^2)*(d39^2-d19^2);

% Augmented Matrix
A = [a11 a12 a13 a14 a15 a16
     a21 a22 a23 a24 a25 a26
     a31 a32 a33 a34 a35 a36
     a41 a42 a43 a44 a45 a46
     a51 a52 a53 a54 a55 a56
     a61 a62 a63 a64 a65 a66];
```

```
% Just for confirmation. Never Used
Matrix = A;



% Result
B = [b1
     b2
     b3
     b4
     b5
     b6];

 % mldivide function
x = A\B;
% x = pinv(A)*B;

%%%%%%%%%%%%%%%%%%%% Unknowen Inner Distances %%%%%%%%%%%%%%%%%5
% Distance Between S1 - S2
d12 = sqrt(x(3)/(1-x(4)-x(5)));
% Distance Between S1 - S3
d13 = sqrt((x(3)*x(5))/(1-x(4)-x(5)));
% Distance Between S2 - S3
d23 = sqrt((x(3)*x(4))/(1-x(4)-x(5)));

 % Just for confirmation. Never Used
Unknown_Dist = [d12 d13 d23];

%%%%%%%%%%%%%%%%%% Final Coordinates %%%%%%%%%%%%%%%%%%%%%
y2 = d12;
y3 = (d12^2+d13^2-d23^2)/(2*d12);
x3 = sqrt(d13^2-((d12^2+d13^2-d23^2)/(2*d12))^2);

% Just for Representation
S  = [ 0  0 0
       0 y2 0
      x3 y3 0];

SS1 = [0 0 0];
SS2 = [0 y2 0];
SS3 = [x3 y3 0];




% Distance between errored S1 and actual S1
DD1 = pdist2(S1,SS1,'euclidean');
% Distance between errored S1 and actual S1
DD2 = pdist2(S2,SS2,'euclidean');
% Distance between errored S1 and actual S15
DD3 = pdist2(S3,SS3,'euclidean');

DDD = [DD1 DD2 DD3];

x1 = 0;
y1 = 0;
```

```
z1 = 0;
x2 = 0;



z2 = 0;
z3 = 0;
figure,
scatter3(150 , 90, 50,'d','filled');
hold on, scatter3(-100 , -120, 50,'d','filled');



hold on, scatter3(-80 , 130, 50,'d','filled');
hold on, scatter3(140 , -70, 50,'d','filled');
hold on, scatter3(60 , 120, 50,'d','filled');
hold on, scatter3(-90 , -120, 50,'d','filled');
hold on, scatter3(x1,y1,z1,'*');
hold on; scatter3(x2,y2,z2,'*');
hold on; scatter3(x3,y3,z3,'*');
```